Midterm Examination
NE-630: APPLIED REACTOR THEORY

PART A: Closed books and notes. Hand in before beginning Part B.

1. Define in words the physical meaning or interpretation of the following quantities. [5 points each]
   (a) the one-speed flux density, \( \phi(r) \)
   (b) the radioactive decay constant, \( \lambda \)
   (c) the geometric buckling, \( B_{\text{geom}}^2 \)
   (d) the \( x \)-component of the neutron current density vector, \( J_x(r) \)
   (e) the square of the diffusion length, \( L^2 \)

2. A monodirectional neutron beam of intensity \( 5 \times 10^{10} \) neutrons cm\(^{-2}\) s\(^{-1}\) uniformly irradiates a small sample of uranium. The sample has a volume of \( 10^{-3} \) cm\(^3\), an atom density of \( 4.8 \times 10^{22} \) cm\(^{-3}\), and the following microscopic cross sections: \( \sigma_a = 7.6 \) barns, and \( \sigma_f = 4.2 \) barns. How many fissions occur in the sample during the first five seconds of irradiation? [15 points]
Part B: Open books and notes. Begin only after handing in Part A.

3. For each of the following one-speed, steady-state diffusion problems (i) sketch the problem geometry showing your choice of coordinate system, (ii) write the appropriate form of the diffusion equation for each diffusing region in which this equation holds, (iii) write the general solution for each region including any particular solution, and (iv) write the boundary/source conditions you would use to determine the values of the arbitrary constants in your general solution. \[15 \text{ points each}\]

(a) A homogeneous infinitely-long cylinder of radius $R$ contains a central cylindrical void of radius $R_o < R$. In the diffusing medium of the sphere is a uniformly distributed source of strength $S$ cm$^{-3}$ s$^{-1}$. The cylinder is surrounded by a layer of a perfect absorber of thickness $T$. The cylinder and its outer absorber layer are then embedded in an infinite medium of water.

(b) A bare, homogeneous sphere of radius $R$ contains a volumetric source of strength $S$ neutrons cm$^{-3}$ s$^{-1}$ that is uniformly distributed in the spherical shell $R_1 < r < R$.

(c) An infinite, bare, homogeneous slab of thickness $T$ contains a distributed volumetric source of strength $S(x) = \alpha x^2$ where $\alpha$ is a positive constant and $x$ is measured from the left surface of the slab. The slab is also irradiated with a beam of neutrons of intensity $I_o$ cm$^{-2}$ s$^{-1}$ that is normally incident on the right surface.

4. The one-speed diffusion equation for the steady-state flux density in an infinite homogeneous medium which contains a uniformly distributed source of neutrons ($S_o$ neutrons cm$^{-3}$ s$^{-1}$) can be written as

$$D\nabla^2 \phi - \Sigma_a \phi + \nu \Sigma_f \phi + S_o = 0.$$  

(a) Explain why $\nabla^2 \phi$ vanishes in this particular problem. \[5 \text{ points}\]

(b) Calculate an expression for the flux density. \[5 \text{ points}\]

(c) For what values of $D$, $\Sigma_a$, and $\nu \Sigma_f$ is your expression for the flux density valid? Explain your reasoning. \[5 \text{ points}\]