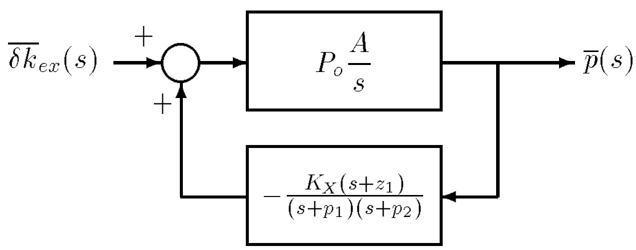
NE-696: NUCLEAR SYSTEMS DESIGN
Final Examination

May 13, 1997

Open books and Notes

The system diagram for a linearized point reactor *without* any temperature feedback but *with* xenon feedback can be drawn as follows.



$$
\begin{aligned}
A &> 0 \\
K_X &= \frac{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o}{\lambda_x + \sigma_x \phi_o} \left(\frac{\sigma_x}{\kappa \beta \Sigma_a} \right) \\
z_1 &= \frac{\lambda_i \lambda_x (\gamma_i + \gamma_x)}{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o} \\
p_1 &= \lambda_i \\
p_2 &= \lambda_x + \sigma_x \phi_o
\end{aligned}
$$

1. Sketch how K_X, z_1, p_1, and p_2 vary with ϕ_o. *[5 points]*
2. Show that the reactor zero power transfer function can be written as $P_o Z(s) \simeq A/s$ for the low frequencies of concern with xenon instabilities. Give an explicit expression for the constant A. *[10 points]*
3. Show that the characteristic equation for the closed-loop transfer function has the form
$$s^3 + a_1 s^2 + a_2 s + a_3 = 0.$$
Give explicit expressions for the coefficients a_i. *[10 points]*
4. Construct the Routh array for this system and determine from it the stability of the closed-loop system. Consider the case $\phi_o < (\phi_o)_{crit}$ and the case $\phi_o > (\phi_o)_{crit}$. Here $(\phi_o)_{crit} \equiv (\gamma_x \lambda_x)/(\gamma_i \sigma_x)$. *[15 points]*
5. Write an expression for the open-loop transfer function, and sketch the root locus diagrams for the case $\phi_o < (\phi_o)_{crit}$ and for the case $\phi_o > (\phi_o)_{crit}$. What can you say about the stability of the system for these two cases? *[15 points]*
6. Sketch the Bode gain and phase plots for the open-loop transfer function for the case $\phi_o < (\phi_o)_{crit}$ and for the case $\phi_o > (\phi_o)_{crit}$. *[15 points]*
7. Draw the Nyquist diagram for this reactor feedback system for the case $\phi_o < (\phi_o)_{crit}$ and for the case $\phi_o > (\phi_o)_{crit}$. What can you say about the stability of each case? Remember to use negative feedback. *[15 points]*
8. For the cases when the system is unstable, derive an expression for the frequency ω_o of the unstable power fluctuations. In terms of this frequency, under what conditions is the system unstable? *[15 points]*