The system diagram for a linearized point reactor without any temperature feedback but with xenon feedback can be drawn as follows.

\[ A > 0 \]

\[ K_X = \frac{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o}{\lambda_x + \sigma_x \phi_o} \left( \frac{\sigma_x}{\kappa \beta \Sigma_a} \right) \]

\[ z_1 = \frac{\lambda_i \lambda_x (\gamma_i + \gamma_x)}{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o} \]

\[ p_1 = \lambda_i \]

\[ p_2 = \lambda_x + \sigma_x \phi_o \]

1. Sketch how \( K_X, z_1, p_1, \) and \( p_2 \) vary with \( \phi_o \). [5 points]

2. Show that the reactor zero power transfer function can be written as \( P_0 Z(s) \sim A/s \) for the low frequencies of concern with xenon instabilities. Give an explicit expression for the constant \( A \). [10 points]

3. Show that the characteristic equation for the closed-loop transfer function has the form

\[ s^3 + a_1 s^2 + a_2 s + a_3 = 0. \]

Give explicit expressions for the coefficients \( a_i \). [10 points]

4. Construct the Routh array for this system and determine from it the stability of the closed-loop system. Consider the case \( \phi_o < (\phi_o)_{crit} \) and the case \( \phi_o > (\phi_o)_{crit} \). Here \( (\phi_o)_{crit} \equiv (\gamma_x \lambda_x) / (\gamma_i \sigma_x) \). [15 points]

5. Write an expression for the open-loop transfer function, and sketch the root locus diagrams for the case \( \phi_o < (\phi_o)_{crit} \) and for the case \( \phi_o > (\phi_o)_{crit} \). What can you say about the stability of the system for these two cases? [15 points]

6. Sketch the Bode gain and phase plots for the open-loop transfer function for the case \( \phi_o < (\phi_o)_{crit} \) and for the case \( \phi_o > (\phi_o)_{crit} \). [15 points]

7. Draw the Nyquist diagram for this reactor feedback system for the case \( \phi_o < (\phi_o)_{crit} \) and for the case \( \phi_o > (\phi_o)_{crit} \). What can you say about the stability of each case? Remember to use negative feedback. [15 points]

8. For the cases when the system is unstable, derive an expression for the frequency \( \omega_o \) of the unstable power fluctuations. In terms of this frequency, under what conditions is the system unstable? [15 points]