Final Examination

NE-630: APPLIED REACTOR THEORY

PART A: Closed books and notes. Hand in before beginning Part B.

1. Match each item on the left with the letter of an item on the right which gives the closest relationship. [15 points]

   (a) $J^+_T$  
   (b) 0.0693/\(\lambda\)  
   (c) \(q(r, E)\)  
   (d) \(L^+_T + \tau_T\)  
   (e) \(\frac{\Sigma_a^f}{(\Sigma_a^f + \Sigma_a^M)}\)  
   (f) \(\nabla^2 \phi(r, \tau) = \frac{\partial q(r, \tau)}{\partial \tau}\)  
   (g) \(\nabla \cdot \mathbf{J}(r) = -D \nabla^2 \phi(r)\)  
   (h) \((1 + B^2 L^2_T)^{-1}\)  
   (i) \(D(E)/\Sigma_o(E)\)  
   (j) \(v n(r)\)  
   (k) \(n\)  
   (l) \(a E\)  
   (m) \(G(r', \mathbf{r}''') = G(\mathbf{r}'', \mathbf{r}')\)  
   (n) \(\xi\)  
   (o) \(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr}\)  

(a) leakage rate  
(b) thermal fission factor  
(c) diffusion length  
(d) BNL-325  
(e) slowing down density  
(f) reciprocity theorem  
(g) average energy loss per collision  
(h) Fermi age  
(i) mean travel distance between collisions  
(j) average gain in lethargy per scatter  
(k) Laplacian in cylindrical geometry  
(l) neutron scattering rate per unit volume  
(m) thermal neutron flux density  
(n) transport equation  
(o) thermal nonleakage probability  
(p) flux density  
(q) half-life  
(r) thermal utilization factor  
(s) Fick’s law  
(t) Laplacian in spherical geometry  
(u) thermal diffusion length  
(v) flow per unit area in the positive z-direction  
(w) Fermi-age equation  
(x) migration area  
(y) minimum energy after scattering  
(z) fast nonleakage probability

2. A homogeneous material has the following nuclear properties: \(\nu = 2.50, \Sigma_a = 0.13 \text{ cm}^{-1}, \Sigma_f = 0.075 \text{ cm}^{-1}, \rho = 0.65, \text{ and } \epsilon = 1.022\). Can a critical bare core be created from this material? Explain your reasoning. [10 points]
3. Define in words the physical meaning of each factor in the four-factor formula $k_\infty = \epsilon \eta f$. 
[10 points]
Part B: Open books and notes. Begin only after handing in Part A.

Note: Do Problems 4 and 5, and either Problem 6 or 7.

4. For the following one-speed, steady-state diffusion problem (i) sketch the geometry showing your coordinate system, (ii) write the appropriate form of the diffusion equation for each region in which this equation holds, (iii) write the general solution for each region including any particular solution, and (iv) write the boundary/source conditions you would use to determine the values of the arbitrary constants in your general solution. [15 points]

(a) An infinite homogeneous slab, composed of a diffusing material, has a thickness $2T$. The slab is irradiated uniformly on its right surface by a perpendicularly incident neutron beam of intensity $I_0$ cm$^{-2}$ s$^{-1}$. The slab contains a volumetric source $S(x)$ which varies with the distance $x$ from the left surface as

$$S(x) = \begin{cases} 
S_o x e^{\alpha x} & \text{for } 0 < x < T \\
0 & \text{for } T < x < 2T
\end{cases}$$

where $\alpha$ is a constant. Finally, at the center of the slab there is an infinite plane isotropic neutron source of strength $S_p$ cm$^{-2}$ s$^{-1}$. Assume the slab is surrounded by vacuum.

5. A bare critical core is to be created by dissolving highly enriched uranyl sulfate ($^{235}$UO$_2$SO$_4$) into a water-filled cubic tank 80 cm on a side.

(a) Explain why $(p \approx 1$ for this mixture. [5 points]
(b) Calculate the critical moderator-to-fuel ratio. Assume room temperature. [10 points]
(c) Calculate the mass (in kg) of the $^{235}$U needed for criticality. [10 points]
(d) Estimate $k_{eff}$ of the core formed by deforming the bare critical core into a sphere of the same volume. [10 points]

6. At a French fuel reprocessing plant, an aqueous solution of fully-enriched uranyl sulfate is to be transferred through, or stored temporarily in, long cylindrical pipes. The maximum expected concentration is 100 g/liter. (a) What is the maximum value of $k_{\infty}$ that can be expected for the solution? (b) What is the maximum pipe radius that can safely be used to avoid a criticality accident? Be sure to state any assumptions used. [15 points]

7. Consider a bare sphere of radius $R_2$ that contains a central region of radius $R_1$. The central region is filled with a perfect absorber, and the remainder of the sphere is composed of a homogeneous multiplying material whose material buckling is denoted by $B$ ($>0$). Assuming one-speed diffusion theory, show that the critical condition for a non-trivial solution of the buckling equation is

$$\tan BR_2 = \tan BR_1.$$ 

From this result show that the critical dimensions are

$$R_2 - R_1 = \frac{\pi}{B}.$$ 

For simplicity assume $R_1$ and $R_2$ both include the extrapolation distance (or, equivalently, the extrapolation distance is negligible compared to $R_1$ and $R_2$). [15 points]