1. Consider a source-free reactor operating at a steady power level of $P_o$. At $t = 0$, the reactivity is varied in such a way as to cause the reactor power to vary as

$$P(t) = P_o + P_o e^{\alpha t}, \quad t > 0.$$ 

Here $\alpha$ a positive constant. Find and sketch the reactivity as a function of time that would produce this power transient. Assume a one delayed-neutron group model and neglect any feedback effects. [15 points]

2. The open-loop transfer function for a negative feedback system is

$$G(s)H(s) = \frac{K}{(s + 2)(s + \alpha)}.$$ 

(a) What is the characteristic equation for this system? [5 points]
(b) On a plot of $K$ versus $\alpha$, indicate the region in which the closed-loop system is stable. [5 points]
(c) Sketch the root locus diagram for positive $K$ when $\alpha = -1$. For what values of $K$ is this system stable? [5 points]

3. Consider the following feedback system.

![Feedback System Diagram]

(a) Construct the Routh array for the closed-loop system and determine a minimal set of necessary and sufficient conditions on the parameters $a$, $b$, and $K$ to ensure an asymptotically stable system. [5 points]
(b) Show that for stability $a$ and $b$ must both be non-negative. [5 points]
(c) Construct the root locus diagram for this system when $a > 0$, $b > 0$ and $K < 0$. [5 points]
(d) Between what two values of $K$ is the system stable? [5 points]
4. A positive feedback system has the following open-loop transfer function.

\[ G(s)H(s) = \frac{K(s + a)}{s^2(s + b)}, \quad a, b > 0 \]

Draw the root locus diagram for (a) \( K < 0 \) and \( a > b \), (b) \( K < 0 \) and \( b > a \), (c) \( K > 0 \) and \( a > b \), and (d) \( K > 0 \) and \( b > a \). Indicate which case(s) produce an asymptotically stable system. [15 points]

5. In class the following model was developed for temperature and xenon feedback.

Recall that both \( K_X \) and \( z_1 \) become negative when \( \phi_o > (\phi_o)_{crit} \equiv (\gamma_x \lambda_x)/(\gamma_i \sigma_x) \).

(a) Sketch how \( K_X \), \( z_1 \), \( p_1 \), and \( p_2 \) vary with \( \phi_o \). [2 points]

(b) Sketch a root locus diagram for the closed-loop system for the case \( \phi_o < (\phi_o)_{crit} \) and for the case \( \phi_o > (\phi_o)_{crit} \). [8 points]

(c) Sketch the Bode gain and phase plots for this open-loop transfer function for the case \( \phi_o < (\phi_o)_{crit} \) and for the case \( \phi_o > (\phi_o)_{crit} \). [10 points]

(d) Assuming negative feedback, draw the Nyquist diagram for this reactor feedback system for the case \( \phi_o < (\phi_o)_{crit} \) and for the case \( \phi_o > (\phi_o)_{crit} \). What can you say about the stability of each case? [15 points]