NE-696: NUCLEAR SYSTEMS DESIGN
Final Examination

May 5, 1995

Open books and Notes

1. Consider a source-free reactor operating at a steady power level of P_o. At $t = 0$, the reactivity is varied in such a way as to cause the reactor power to vary as

$$P(t) = P_o + P_o e^{\alpha t}, \qquad t > 0.$$

Here α a positive constant. Find and sketch the reactivity as a function of time that would produce this power transient. Assume a one delayed-neutron group model and neglect any feedback effects. *[15 points]*

2. The open-loop transfer function for a *negative* feedback system is

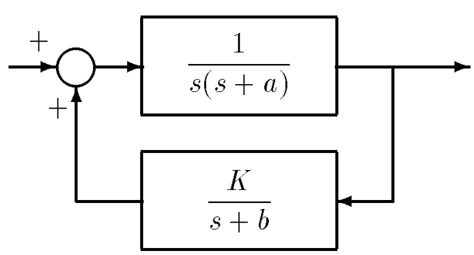
$$G(s)H(s) = \frac{K}{(s+2)(s+\alpha)}.$$

(a) What is the characteristic equation for this system? *[5 points]*

(b) On a plot of K versus α, indicate the region in which the closed-loop system is stable. *[5 points]*

(c) Sketch the root locus diagram for positive K when $\alpha = -1$. For what values of K is this system stable? *[5 points]*

3. Consider the following feedback system.



(a) Construct the Routh array for the closed-loop system and determine a minimal set of necessary and sufficient conditions on the parameters a, b, and K to ensure an asymptotically stable system. *[5 points]*

(b) Show that for stability a and b must both be non-negative. *[5 points]*

(c) Construct the root locus diagram for this system when $a > 0$, $b > 0$ and $K < 0$. *[5 points]*

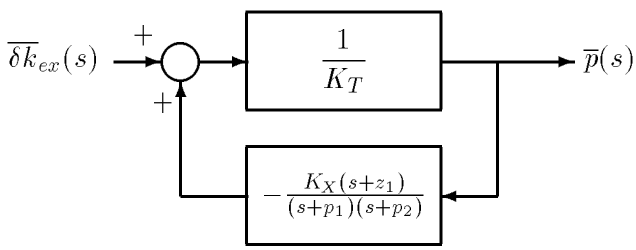
(d) Between what two values of K is the system stable? *[5 points]*

4. A positive feedback system has the following open-loop transfer function.

$$G(s)H(s) = \frac{K(s+a)}{s^2(s+b)}, \qquad a, b > 0$$

Draw the root locus diagram for (a) $K < 0$ and $a > b$, (b) $K < 0$ and $b > a$, (c) $K > 0$ and $a > b$, and (d) $K > 0$ and $b > a$. Indicate which case(s) produce an asymptotically stable system. *[15 points]*

5. In class the following model was developed for temperature and xenon feedback.



$$\begin{aligned} K_T &> 0 \\ K_X &= \frac{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o}{\lambda_x + \sigma_x \phi_o} \left(\frac{\sigma_x}{\kappa \beta \Sigma_a} \right) \\ z_1 &= \frac{\lambda_i \lambda_x (\gamma_i + \gamma_x)}{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o} \\ p_1 &= \lambda_i \\ p_2 &= \lambda_x + \sigma_x \phi_o \end{aligned}$$

Recall that both K_X and z_1 become negative when $\phi_o > (\phi_o)_{crit} \equiv (\gamma_x \lambda_z)/(\gamma_i \sigma_x)$.

(a) Sketch how K_X, z_1, p_1, and p_2 vary with ϕ_o. *[2 points]*

(b) Sketch a root locus diagram for the closed-loop system for the case $\phi_o < (\phi_o)_{crit}$ and for the case $\phi_o > (\phi_o)_{crit}$. *[8 points]*

(c) Sketch the Bode gain and phase plots for this open-loop transfer function for the case $\phi_o < (\phi_o)_{crit}$ and for the case $\phi_o > (\phi_o)_{crit}$. *[10 points]*

(d) Assuming negative feedback, draw the Nyquist diagram for this reactor feedback system for the case $\phi_o < (\phi_o)_{crit}$ and for the case $\phi_o > (\phi_o)_{crit}$. What can you say about the stability of each case? *[15 points]*