Open books and Notes

1. Consider a system whose transfer function is $G(s) = 1/[(s + 1)(s + 2)]$. (a) Calculate and sketch the unit impulse response $z(t)$ for this system. (b) What is this system’s asymptotic response $P_{as}(t)$ to an input of $A \sin(\omega t)$. [10 points]

2. In the following feedback system the constant $b$ is a positive fixed number, while $a$ can be any positive value and $A$ can have any positive or negative value.

$$\begin{align*}
\begin{array}{c}
+ \\
\end{array} & \quad \begin{array}{c}
- \\
\end{array} \\
\begin{array}{c}
A(s + a) \\
- \frac{1}{s - b}
\end{array} & \quad \begin{array}{c}
\frac{1}{s} \\
\end{array}
\end{align*}$$

Construct the characteristic polynomial for this system and determine for what values of $A$ and $a$ the system will be asymptotically stable. Show on a plot of $A$ versus $a$ the region of stability. [10 points]

3. The open-loop transfer function for a negative feedback system can be written as

$$G(s)H(s) = \frac{K + a}{(s - 1)(s + 2)(s + a)}.$$ 

Use the Routh-Hurwitz method to determine conditions on $a$ and $K$ for the closed-loop system to be stable. [15 points]

4. Consider the following reactor feedback model.

$$\begin{align*}
\begin{array}{c}
\overline{\kappa}_{e_{e}}(s) \\
\end{array} & \quad \begin{array}{c}
+ \\
\end{array} \\
\begin{array}{c}
\frac{10}{s} \\
\end{array} & \quad \begin{array}{c}
\frac{K(s - 10)}{(s + 10)} \\
\end{array}
\end{align*}$$

(a) Sketch root locus diagrams for $K > 0$ and for $K < 0$. [10 points]

(b) Construct the characteristic equation for this system and determine the value of $K$ at which the reactor becomes unstable. [10 points]

(c) What is the frequency of the power oscillations at the threshold of instability? [5 points]

(d) Sketch Nyquist diagrams for this system for $K > 0$ and for $K < 0$. [10 points]
5. As derived in class, the open-loop transfer function of a reactor with both temperature and xenon feedback can be written as

\[ G(s)H(s) = \frac{K_X}{K_T} \frac{(s + z_1)}{(s + p_1)(s + p_2)}, \quad K_T > 0 \]

where

\[ K_X \equiv \frac{\sigma_x}{\kappa \beta \Sigma_{ao}} \left[ \frac{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o}{\lambda_x + \sigma_x \phi_o} \right] \]

\[ z_1 \equiv \frac{\lambda_i \lambda_x (\gamma_i + \gamma_x)}{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o} \]

\[ p_1 \equiv \lambda_i \]

\[ p_2 \equiv \lambda_x + \sigma_x \phi_o \]

Recall that both \( K_X \) and \( z_1 \) become negative when \( \phi_o > (\phi_o)_{crit} \equiv (\gamma_x \lambda_x)/(\gamma_i \sigma_x) \).

(a) Sketch how \( K_X, z_1, p_1, \) and \( p_2 \) vary with \( \phi_o \). [5 points]

(b) Sketch the Bode gain and phase plots for this open-loop transfer function for the case \( \phi_o < (\phi_o)_{crit} \) and for the case \( \phi_o > (\phi_o)_{crit} \). [10 points]

(c) Assuming negative feedback, draw the Nyquist diagram for this reactor feedback system for the case \( \phi_o < (\phi_o)_{crit} \) and for the case \( \phi_o > (\phi_o)_{crit} \). What can you say about the stability of each case? [5 points]