NE-696: NUCLEAR SYSTEMS DESIGN Final Examination

May 7, 1994

Open books and Notes

- 1. Consider a system whose transfer function is G(s) = 1/[(s+1)(s+2)]. (a) Calculate and sketch the unit impulse response z(t) for this system. (b) What is this system's asymptotic response $P_{as}(t)$ to an input of $A \sin(\omega t)$. [10 points]
- 2. In the following feedback system the constant b is a positive fixed number, while a can be any positive value and A can have any positive or negative value.



Construct the characteristic polynomial for this system and determine for what values of A and a the system will be asymptotically stable. Show on a plot of A versus a the region of stability. [10 points]

3. The open-loop transfer function for a negative feedback system can be written as

$$G(s)H(s) = \frac{K+\alpha}{(s-1)(s+2)(s+\alpha)}$$

Use the Routh-Hurwitz method to determine conditions on α and K for the closed-loop system to be stable. [15 points]

4. Consider the following reactor feedback model.



- (a) Sketch root locus diagrams for K > 0 and for K < 0. [10 points]
- (b) Construct the characteristic equation for this system and determine the value of *K* at which the reactor becomes unstable. [10 points]
- (c) What is the frequency of the power oscillations at the threshold of instability? [5 points]
- (d) Sketch Nyquist diagrams for this system for K > 0 and for K < 0. [10 points]

5. As derived in class, the open-loop transfer function of a reactor with both temperature and xenon feedback can be written as

$$G(s)H(s) = \frac{K_X}{K_T} \frac{(s+z_1)}{(s+p_1)(s+p_2)}, \quad K_T > 0$$

where

$$K_X \equiv \frac{\sigma_x}{\kappa \beta \Sigma_{ao}} \left[\frac{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o}{\lambda_x + \sigma_x \phi_o} \right]$$
$$z_1 \equiv \frac{\lambda_i \lambda_x (\gamma_i + \gamma_x)}{\gamma_x \lambda_x - \gamma_i \sigma_x \phi_o}$$
$$p_1 \equiv \lambda_i$$
$$p_2 \equiv \lambda_x + \sigma_x \phi_o$$

Recall that both K_X and z_1 become negative when $\phi_o > (\phi_o)_{crit} \equiv (\gamma_x \lambda_z)/(\gamma_i \sigma_x)$.

- (a) Sketch how K_X , z_1 , p_1 , and p_2 vary with ϕ_o . [5 points]
- (b) Sketch the Bode gain and phase plots for this open-loop transfer function for the case $\phi_o < (\phi_o)_{crit}$ and for the case $\phi_o > (\phi_o)_{crit}$. [10 points]
- (c) Assuming negative feedback, draw the Nyquist diagram for this reactor feedback system for the case $\phi_o < (\phi_o)_{crit}$ and for the case $\phi_o > (\phi_o)_{crit}$. What can you say about the stability of each case? [15 points]