1. Consider a source-free reactor operating at a steady power level of $P_o$. At $t = 0$, the reactivity is varied in such a way as to cause the reactor power to exponentially decrease as

$$ P(t) = P_o \exp(-\lambda t), \quad t > 0. $$

Here $\lambda$ is the decay constant for the delayed neutrons (assume a one delayed-neutron group in your analysis). Find and sketch the reactivity as a function of time that would produce this power transient. Neglect any feedback effects. (15 points)

2. Sketch the Bode plots for both positive and negative values of $K$ for the following transfer function

$$ G(s)H(s) = \frac{K(s + 1)(s + 100)}{(s + 10)^2(s + 1000)}. $$

Sketch two feedback systems for which this would be the closed-loop transfer function. (10 points)

3. Consider the following system with a constant feedback gain $K (> 0)$

\[ \begin{array}{c}
\begin{array}{c}
\text{+} \quad G(s) \quad K
\end{array}
\end{array} \]

in which the subsystem denoted by $G(s)$ is constructed as

\[ \begin{array}{c}
\begin{array}{c}
\frac{1}{s} \quad \frac{1}{s+2} \quad \frac{s+8}{(s+2)^2} \quad \frac{3}{s+6}
\end{array}
\end{array} \]

Sketch the root locus diagram for this system and estimate the value of $K$ for the onset of instability. (20 points)
4. Construct the Nyquist diagram for and determine the stability of a system with negative feedback for which the closed-loop transfer function is given by

\[ G(s)H(s) = \frac{100(s + 10)}{s^2(s + 100)}. \]

Is this system stable? (15 points)

5. Consider a reactor with xenon feedback but for which there is no temperature feedback. (You knew we had to have a xenon problem).

(a) Show that for low frequencies the zero power transfer function \( Z(s) \approx \frac{A}{s} \) where the constant \( A > 0 \). Obtain an expression for \( A \). (4 points)

(b) With the xenon reactivity feedback transfer function derived in class, the reactor at low frequencies can thus be modeled as

\[
\begin{align*}
\text{+} & \quad A/s \\
\text{+} & \quad K_X(s+z_i) \\
\text{–} & \quad \frac{s+p_i}{(s+p_2)(s+p_2)}
\end{align*}
\]

What is the characteristic equation for this system? (4 points)

(c) Construct the Routh array for this system, and derive a criterion on \( AK_X \) for stability. (8 points)

(d) Sketch root locus diagrams for this system as \( AK_X \) varies in magnitude, one for \( \phi_o < (\phi_o)_{crit} \), and one for \( \phi_o < (\phi_o)_{crit} \). Indicate how the root locus diagrams change as \( \phi_o \) increases. What can you say about the stability of this reactor from each diagram? (8 points)

(e) Sketch pairs of Bode plots of the open-loop transfer function for this system, one pair for \( \phi_o < (\phi_o)_{crit} \), and one pair for \( \phi_o < (\phi_o)_{crit} \). (8 points)

(f) Sketch Nyquist diagrams for this system, one for \( \phi_o < (\phi_o)_{crit} \), and one for \( \phi_o < (\phi_o)_{crit} \). What can you say about the stability of this reactor from each diagram? (8 points)